Homework 10

Introduction to Topology, Spring 2023

Due April 10, 2023 at 11:59pm

1. If $E \to B$ and $E' \to B'$ are covering maps, show that $E \times E' \to B \times B'$ is a covering map.
2. If $B' \subseteq B$ is a subspace, and we let $E' = p^{-1}(B')$, show that $p : E' \to B'$ is a covering map.
3. Show that the map

$$\begin{aligned} S^1 &\to S^1 \\ x &\mapsto x^n \end{aligned}$$

   is a covering space for $n > 0$. Here we are viewing $S^1 \subseteq \mathbb{C}$ and doing multiplication in $\mathbb{C}$, which is another way of saying rotation on S^1.
4. Suppose $p : E \to B$ is a covering map, and suppose that B is connected. Suppose there exists a point $b_0 \in B$ such that $p^{-1}(b_0)$ is a finite set with n elements. Show that $p^{-1}(b)$ is a finite set with n elements for all $b \in B$.
5. Since $\mathbb{R} \to S^1$ is a covering map, so is $\mathbb{R} \times \mathbb{R}_{>0} \to S^1 \times \mathbb{R}_{>0}$. There is a homeomorphism $S^1 \times \mathbb{R}_{>0} \cong \mathbb{R}^2 \setminus \{0\}$, so we get a covering map $R \times \mathbb{R}_{>0} \to \mathbb{R}^2 \setminus \{0\}$. Find liftings of the following three paths from $I \to \mathbb{R}^2 \setminus \{0\}$ along this covering map:
   (a) $f(t) = (2 - t, 0)$
   (b) $g(t) = ((1 + t)\cos 2\pi t, (1 + t)\sin 2\pi t)$
   (c) $h(t) = f * g$, where $*$ as usual denotes concatenation of paths.
6. A map $X \to Y$ should give a map $\pi_1(X, x_0) \to \pi_1(Y, y_0)$. We have continuous maps

$$\begin{aligned} \varphi_n : S^1 &\to S^1 \\ x &\mapsto x^n \end{aligned}$$

   for $n \in \mathbb{Z}$. These give rise to maps $\pi_1(S^1, *) \to \pi_1(S^1, *)$. Compute these maps.