Homework 11

Introduction to Topology, Spring 2023

Due April 17, 2023 at 11:59pm

1. Prove the Eckmann–Hilton theorem (Proposition 11.2.3 in the notes).

2. We don't know $\pi_i(S^n)$ for $i > n$ in general. But show that $\pi_i(S^1) = 0$ for all $i > 1$. You may assume the following general fact about covering spaces:

   Proposition 0.0.1. *Suppose $p : E \to B$ is a covering map and fix a point $e_0 \in E$ and let $b_0 := p(e_0)$. Fix Y a path-connected topological space which is also locally path-connected[1]. Suppose $f : Y \to B$ is a continuous map and suppose there is a point $y_0 \in Y$ such that $f(y_0) = b_0$.*

   If $f_*(\pi_1(Y, y_0)) \subset p_*(\pi_1(E, e_0))$, then there exists a map $\widetilde{f} : Y \to E$ satisfying $\widetilde{f}(y_0) = e_0$ and $p \circ \widetilde{f} = f$. This is summarized in the diagram

$$\begin{array}{ccc} & & E \\ & \overset{\widetilde{f}}{\nearrow} & \downarrow p \\ Y & \xrightarrow[f]{} & B \end{array}$$

3. By analogy with the case of S^1, show that if X is path-connected and $x_0, x_1 \in X$ then there exists a group isomorphism $\pi_n(X, x_0) \cong \pi_n(X, x_1)$.

[1] A space is locally path-connected if for every point $x \in X$ and every open set U containing x there exists a path-connected open set $V \subseteq U$ such that $x \in V$. You may assume that spheres are path-connected and locally path-connected