Week 10 Group Problems — Topology — Spring 2023

Let's do a few covering space examples.

1. Prove that the map $\mathbb{C} \to \mathbb{C}$ given by $z \mapsto z^2$ is not a covering map.

2. Recall that the *real projective line* $\mathbb{RP}^1$ is the quotient $\mathbb{R}^2 - \{0\} / \sim$ where two points are equivalent if they are real (possibly negative) scalar multiples of each other. Convince yourself that the map

$$\pi : S^1 \to \mathbb{RP}^1$$

is a covering map. What is the size of a fiber $\pi^{-1}(b_0)$? Bonus for later: can you generalize this to S^n, the n-sphere in $n + 1$-dimensional space?

3. Use the fact that $\mathbb{R} \to S^1$ is a covering map (as shown in class) to show that $\mathbb{C} \to \mathbb{C} - \{0\}$ taking $z \mapsto e^z$ is a covering map.

4. Draw a figure eight. Its fundamental group is the *free group on two generators*; this is the group whose elements are products of a, b, a^{-1} and b^{-1} (for example $a^3ba^2b^{-4}$ is an element) but which is such that $ab \neq ba$, so the group is not commutative.

   Can you find a universal cover for the figure eight?