Week 6 Group Problems — Topology — Spring 2023

A B C D E F G H I

J K L M N O P Q R

S T U V W X Y Z

Group these letters together into homeomorphism classes. You don't need to precisely parametrize each letter and write down explicit homeomorphisms between ones that you think are homeomorphic, but on the other hand, you should be able to convincingly argue when two letters are homeomorphic, and when two are not.

Note: the letters you are looking at appear to have some width to them. This is of course necessary for you to be able to see them, but if you like you can imagine they are drawn with infinitesimally thick lines. It doesn't matter which way you think about it; do whichever you find more intuitive or useful.

Here are some additional problems, if you finish the classification.

1. Show that if X is connected and $f : X \to \mathbb{R}$ is a continuous function, then $f(X)$ is an interval.

2. A map of topological spaces $f : X \to Y$ is called *locally constant* if for every $x \in X$ there exists an open set U containing x such that f is constant on U; in other words, $f(x) = f(x')$ for all other $x' \in U$.

   (a) Show that if X is connected then f is actually constant.

   (b) We know that $\mathbb{Q} \subseteq \mathbb{R}$ is not connected. Can you write down a locally constant function $f : \mathbb{Q} \to \mathbb{R}$ which is not constant?

   (c) If Y is discrete, then which functions $f : X \to Y$ are locally constant?